系組：化學工程與材料工程學系奈米材料碩士班 甲組　　日期節次：98 年 3 月 21 日第 4 節 15:30-17:00

科目：**工程數學[2931]**

1. (40%) Solve $y(x)$ form the following ordinary differential equation:

   (a) $y''+6y'+9y=0$, *with* $y(0)=a_1$ *and* $y'(0)=-3a_1+a_2$ (10%)

   (b) $2xyy'+y^2=4$, *with* $y(1)=0$ (10%)

   (c) $x^2y''-12y=24x$ (10%)

   (d) $y''+4y'+4y=6e^{-2x}$ (10%)

2. (15%) Use the Laplace transform to solve the differential equations ONLY!!!

$$\begin{cases}\dfrac{dX(t)}{dt}=-aY(t)\\ \dfrac{dY(t)}{dt}=4aX(t)\end{cases}$$ and $t=0, X=X(0), Y=Y(0), a\in R$ 用其他方法不以計分!!!

3. (20%) Define $\boldsymbol{M}=\begin{bmatrix}-1 & 0 & 12 & 0\\ 0 & -1 & 0 & 12\\ 0 & 0 & -1 & -4\\ 0 & 0 & -4 & -1\end{bmatrix}$ and $\boldsymbol{b}=\begin{bmatrix}\frac{9}{8}\\ -\frac{9}{8}\\ \frac{3}{8}\\ -\frac{3}{8}\end{bmatrix}$.

   (a) Find the four eigenvalues (λ_1, λ_2, λ_3 and λ_4) of matrix $\boldsymbol{M}$. (4%)
   (b) Assume $\lambda_1>\lambda_2=\lambda_3>\lambda_4$. Find the two eigenvectors of the matrix $\boldsymbol{M}$, which eigenvalues are λ_1 and λ_4. (8%)
   (c) $\boldsymbol{M}^{-1}$ is the inverse matrix of $\boldsymbol{M}$..Calculate the $\boldsymbol{M}^{-1}\boldsymbol{b}$=? (8%)

4. (25%) Using the Fourier series to expand the following function:

   (a) $f(x)=\cos(ax)$, *with* $-\pi<x\le\pi$ *and* $a\neq$ integer (15%)

   (b) if x= π , please show that : $\cot(x)=\sum_{n=-\infty}^{\infty}\dfrac{1}{x+n\pi}$, $n=$ integer (10%)

There are some useful formulae.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}, \text{with } -L<x\le L$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,\quad n=0,1,2,3\cdots \qquad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx,\quad n=1,2,3\cdots$$

$$2\cos\alpha\cos\beta=\cos(\alpha-\beta)+\cos(\alpha+\beta)$$

$$\sin(a\pm n)\pi=(-1)^n\sin a\pi, \text{if } n=\text{integer}$$