中國文化大學99學年度碩士班考試入學招生考試

系所組：化學工程與材料工程學系奈米材料碩士班甲組

日期節次：99年3月13日第2節 11:00-12:30

科目：**工程數學**

1. (40%) Solve $y(x)$ for the following ordinary differential equation:

   (a) $y''+3y'+2y=0$ (10%)

   (b) $y''+4y=2\sin 2x$ (10%)

   (c) $x^2y''-5xy'+8y=0$, *with* $y(1)=2$ *and* $y'(1)=6$ (10%)

   (d) $xyy'-y^2=x^2$, *with* $y(2)=0$ (10%) *Hint: use u=y/x*

2. (15%) Derive the Laplace transform for the function $\{\sin kt\}$ is

   $$\mathcal{L}\{\sin kt\}=\frac{k}{s^2+k^2}$$

3. (20%) The eigenvalues of the matrix $M=\begin{bmatrix} a & b & 0 \\ b & a & b \\ 0 & b & a \end{bmatrix}$ are –3, 1 and 5.

   (a) Find the values of a and b (assume: $a>0$ and $b>0$) (10%)

   (b) Find the normalized eigenvectors for matrix M. (10%)

4. (25%) Using the Fourier series to expand the following function:

   (a) $f(x)=x^2$, *with* $-\pi<x\le\pi$ (15%) (b) if x= π , please show that : $\sum_{n=1}^{\infty}\frac{1}{n^2}=?$ (10%)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}, \text{with } -L<x<L$$

$$, a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,\quad n=0,1,2,3\cdots \quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}dx,\quad n=1,2,3\cdots$$

$$\int x^2\sin ax\,dx=\frac{2x}{a^2}\sin ax+(\frac{2}{a^3}-\frac{x^2}{a})\cos ax+C$$

$$\int x^2\cos ax\,dx=\frac{2x}{a^2}\cos ax+(\frac{x^2}{a}-\frac{2}{a^3})\sin ax+C$$